Common Core State Standards for Mathematics

Flip Book
Grade 2

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Planning Advice--Focus on the Clusters

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptually understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom. (www.achievethecore.org)

As the Kansas College and Career Ready Standards (KCCRS) are carefully examined, there is a realization that with time constraints of the classroom, not all of the standards can be done equally well and at the level to adequately address the standards. As a result, priorities need to be set for planning, instruction and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "While the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. … It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)

The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In planning for instruction "grain size" is important. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Daro (Teaching Chapters, Not Lessons—Grain Size of Mathematics), strands are too vague and too large a grain size, while lessons are too small a grain size. About 8 to 12 units or chapters produce about the right "grain size". In the planning process staff should attend to the clusters, and think of the standards as the ingredients of cluster, while understanding that coherence exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions that argue 2 days instead of 3 days on a topic because it is a lower priority detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, lenses focused on lessons can also provide too narrow a view which compromises the coherence value of closely related standards.

The video clip Teaching Chapters, Not Lessons—Grain Size of Mathematics that follows presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with "grain size", clusters have been given **priorities** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they demine distribution of time for both planning and instruction, helping to assure that students really understand before moving on. Each cluster has been given a priority level. As professional staffs begin planning, developing and writing units as Daro suggests, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level by Zimba. The three levels are referred to as:—**Focus**, **Additional** and **Sample**. Furthermore, Zimba suggests that about 70% of instruction should relate to the Focus clusters. In planning, the lower two priorities (Additional and Sample) can work together by supporting the Focus priorities. The advanced work in the high school standards is often found in "Additional and Sample clusters". Students who intend to pursue STEM careers or Advance Placement courses should master the material marked with "+" within the standards. These standards fall outside of priority recommendations.

Recommendations for using cluster level priorities

Appropriate Use:

- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through: sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possibility quality; the additional work of the grade should indeed support the Focus priorities and not detract from it.
- Set priorities for other implementation efforts taking the emphasis into account such as: staff development; new curriculum development; revision of existing formative or summative testing at the state, district or school level.

Things to Avoid:

- Neglecting any of the material in the standards rather than connecting the Additional and Sample clusters to the other work of the grade
- Sorting clusters from Focus to Additional to Sample and then teaching the clusters in order. To do so would remove the coherence of mathematical ideas and miss opportunities to enhance the focus work of the grade with additional clusters.
- Using the clusters' headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise and coherence of the standards (grain size).

Depth Opportunities

Each cluster, at a grade level, and, each domain at the high school, identifies five or fewer standards for in-depth instruction called Depth Opportunities (Zimba, 2011). Depth Opportunities (DO) is a qualitative recommendation about allocating time and effort within the highest priority clusters --the **Focus** level. Examining the Depth Opportunities by standard reflects that some are beginnings, some are critical moments or some are endings in the progressions. The DO's provide a prioritization for handling the uneven grain size of the content standards. Most of the DO's are not small content elements, but, rather focus on a big important idea that students need to develop.

DO's can be likened to the Priorities in that they are meant to have relevance for instruction, assessment and professional development. In planning instruction related to DO's, teachers need to intensify the mode of engagement by emphasizing: **tight focus**, **rigorous reasoning and discussion** and **extended class time devoted to practice and reflection** and have **high expectation for mastery**. (See Appendix Table 7, Depth of Knowledge (DOK)

In this document, Depth Opportunities are highlighted pink in the Standards section. For example:

5.NBT.6 *Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.*

Depth Opportunities can provide guidance for examining materials for purchase, assist in professional dialogue of how best to develop the DO's in instruction and create opportunities for teachers to develop high quality methods of formative assessment.

Standards for Mathematical Practice in Grade 2

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 2 students complete.

Practice	Explanation and Example
1) Make Sense and Persevere in Solving Problems.	Mathematically proficient students in Grade 2 examine problems, can make sense of the meaning of the task, and find an entry point or a way to start the task. Grade 2 students also develop a foundation for problem solving strategies and become independently proficient on using those strategies to solve new tasks. In Grade 2, students' work still relies on concrete manipulatives and pictorial representations as students solve tasks unless the CCSS refers to the word fluently, which denotes mental mathematics. Grade 2 students also are expected to persevere while solving tasks; that is, if students reach a point in which they are stuck, they can reexamine the task in a different way and continue to solve the task. Lastly, mathematically proficient students complete a task by asking themselves the question, "Does my answer make sense?"
2) Reason abstractly and quantitatively.	Mathematically proficient students in Grade 2 make sense of quantities and the relationships while solving tasks. This involves two processes- decontextualizing and contextualizing. In Grade 2, students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, "There are 25 children in the cafeteria and they are joined by 17 more children. Then, if 19 of those children then leave, how many are still there?" Grade 2 students are expected to translate that situation into the equation: $25 + 17 - 19 = ?$ and then solve the task. Students also contextualize situations during the problem solving process. For example, while solving the task above, students can refer to the context of the task to determine that they need to subtract 19 since 19 children leave. The processes of reasoning also apply to Grade 2 as students begin to measure with standard measurement units by determining the length of quantities based on particular units of measure.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in Grade 2 accurately use definitions and previously established solutions to construct viable arguments about mathematics. In Grade 2 during discussions about problem solving strategies, students constructively critique the strategies and reasoning of their classmates. For example, while solving $74 + 18 - 37$, students may use a variety of strategies, and after working on the task, can discuss and critique each other's reasoning and strategies, citing similarities and differences between strategies.

Practice	Explanation and Example
4) Model with mathematics.	Mathematically proficient students in Grade 2 model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. Grade 2 students still will rely on concrete manipulatives and pictorial representations while solving problems, but the expectation is that they will also write an equation to model problem situations. Likewise, Grade 2 students are expected to create an appropriate problem situation from an equation. For example, students are expected to create a story problem for the equation$24 + 17 - 13 = ?$.
5) Use appropriate tools strategically.	Mathematically proficient students in Grade 2 have access to and use tools appropriately. These tools may include place value (base ten) blocks, hundreds number boards, number lines, and concrete geometric shapes (e.g., pattern blocks, 3-d solids). Students should also have experiences with educational technologies, such as calculators and virtual manipulatives that support conceptual understanding and higher-order thinking skills. During classroom instruction, students should have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. For example, while solving $28 + 17$, students can explain why place value blocks are more appropriate than counters.
6) Attend to precision.	Mathematically proficient students in Grade 2 are precise in their communication, calculations, and measurements. In all mathematical tasks, students in Grade 2 communicate clearly, using grade-level appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions. For example, while measuring objects iteratively (repetitively), students check to make sure that there are no gaps or overlaps. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions.
7) Look for and make use of structure.	Mathematically proficient students in Grade 2 carefully look for patterns and structures in the number system and other areas of mathematics. While solving addition and subtraction problems students can apply the patterns of the number system to skip count by 10s off the decade. For example, Grade 2 students are expected to mentally reason that $33 + 21$ is 33 plus 2 tens, which equals 53 and then an addition one which equals 54. While working in the Numbers in Base Ten domain, students work with the idea that 10 ones equal a ten, and 10 tens equals 1 hundred. Further, Grade 2 students also make use of structure when they work with subtraction as missing addend problems, such as $50 - 33 = ?$ can be written as $33 + ? = 50$ and can be thought of as how much more do I need to add to 33 to get to 50?
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in Grade 2 begin to look for regularity in problem structures when solving mathematical tasks. For example, after solving two digit addition problems by decomposing numbers by place ($33 + 25 = 30 + 20 + 3 + 5$), students may begin to generalize and frequently apply that strategy independently on future tasks. Further, students begin to look for strategies to be more efficient in computations, including doubles strategies and making a ten. Lastly, while solving all tasks, Grade 2 students accurately check for the reasonableness of their solutions during, and after completing the task.

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. **Make sense of problems and persevere in solving them.** • Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Can monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Can understand various approaches to solutions. • Continually ask themselves; "Does this make sense?"	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. • What might you change? • Talk me through the steps you've used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize, represent, and show?
2. **Reason abstractly and quantitatively.** • Make sense of quantities and their relationships. • Are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _____related to _____? • What is the relationship between _____and _____? • What does _____mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use? • Could we have used another operation or property to solve this task? Why or why not?
3. **Construct viable arguments and critique the reasoning of others.** • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	• What mathematical evidence would support your solution? How can we be sure that _____? / How could you prove that._____? Will it still work if._____? • What were you considering when._____? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? • What is the same and what is different about._____? • How could you demonstrate a counter-example?
4. **Model with mathematics.** • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the math they know to solve problems in everyday life. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving or revising the model. • Ask themselves, "How can I represent this mathematically?"	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What's an equation or expression that matches the diagram, number line, chart, table? • Where did you see one of the quantities in the task in your equation or expression? • Would it help to create a diagram, graph, table? • What are some ways to visually represent? • What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. **Use appropriate tools strategically.** • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	• What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use: a graph, number line, ruler, diagram, calculator, manipulative? • Why was it helpful to use._____? • What can using a _____ show us, that ___may not? • In what situations might it be more informative or helpful to use._____?
6. **Attend to precision.** • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • Is there a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, properties can you use to explain.____? • How could you test your solution to see if it answers the problem?
7. **Look for and make use of structure.** • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	• What observations do you make about._____? • What do you notice when._____? • What parts of the problem might you eliminate, simplify? • What patterns do you find in._____? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to._____? • In what ways does this problem connect to other mathematical concepts?
8. **Look for and express regularity in repeated reasoning.** • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	• Will the same strategy work in other situations? • Is this always true, sometimes true or never true? • How would we prove that._____? • What do you notice about._____? • What is happening in this situation? • What would happen if._____? • What Is there a mathematical rule for._____? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

Critical Areas for Mathematics in 2nd Grade

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

1. Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
   (NBT.1, NBT.2, NBT.3, NBT.4, NBT.5, NBT.6, NBT.7, NBT.8, NBT.9)

2. Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
   (OA.1, OA.2, OA.3, OA.4)

3. Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.
   (MD.1, MD.2, MD.3, MD.4, MD.5, MD.6)

4. Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.
   (G.1; G.2; G.3)

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.
For more information please visit the Dynamic Learning Maps and Essential Elements website.

Grade 2 Content Standards Overview

Operations and Algebraic Thinking (OA)

- Represents and solves problems involving addition and subtraction
  OA.1
- Add and subtract within 20
  OA.2
- Work with equal groups of objects to gain foundations for multiplication
  OA.3 **OA.4**

Number and Operations in Base Ten (NBT)

- Understand place value .
  NBT.1 **NBT.2** **NBT.3** **NBT.4**
- Use place value understanding and properties of operations to add and subtract .
  NBT.5 **NBT.6** **NBT.7** **NBT.8** **NBT.9**

Measurement and Data (MD)

- Measure and estimate lengths in standard units
  MD.1 **MD.2** **MD.3** **MD.4**
- Relate addition and subtraction to length
  MD.5 **MD.6**
- Work with time and money
  MD.7 **MD.8**
- Represent and interpret data
  MD.9 **MD.10**

Geometry (GE)

- Reason with shapes and their attributes
  G.1 **G.2** **G.3**

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Represent and solve problems involving addition and subtraction.*

Standard: Grade 2. OA.1

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Table 1, Appendix)

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction**.
- *Represent and solve problems involving addition and subtraction* and *Work with addition and subtraction equations* in Grade 1, to *Relate addition and subtraction to length* and *Work with time and money* in Grade 2, and to *Solve problems involving the four operations, and identify and explain patterns in arithmetic* in Grade 3.

Explanation and Examples:

This standard calls for calls for students to add and subtract numbers within 100 in the context of one- and two-step word problems. Students should have ample experiences working on all the subtypes of problems illustrated within Table 1 (See Appendix pg. 66) and that have unknowns in all positions, including:

Results Unknown:	**Change Unknown**:	**Start Unknown**:
There are 29 students on the playground. Then 18 more students showed up. How many students are there now? $29 + 18 = ?$	There are 29 students on the playground. Some more students show up. There are now 47 students. How many students came? $29 + ? = 47$	There are some students on the playground. Then 18 more students came. There are now 47 students. How many students were on the playground at the beginning? $? + 18 = 47$

This standard also calls for students to solve one- and two-step problems using drawings, objects and equations. Students should be able to use place value blocks, hundreds charts, number lines, or create drawings of any of these tools to support their work.

Examples of one-step problems with unknowns in different places are provided in Table 1, Appendix page 66)

Two step-problems include situations where students have to add and subtract within the same problem.

Example:

Major Supporting Additional **Depth Opportunities(DO)**

In the morning there are 25 students in the cafeteria. 18 more students come in. After a few minutes, some students leave. If there are 14 students still in the cafeteria, how many students left the cafeteria? *Write an equation for your problem.*

EXPECT students to use *place value blocks (base 10*), *number line, hundreds chart*, etc. to show, solve and **explain their reasoning**. Just explaining by telling the steps of the procedure will not be enough. Students need to understand the operations and the process. Instead of asking for the "answer", say: "Using "the model," "the relationships," "the structure," or "justify your answer."

Word problems that are connected to students' lives can be used to develop fluency with addition and subtraction. Table 1 describes the four different addition and subtraction situations and their relationship to the position of the unknown.

Example:

- Take From (Result unknown): David had 63 stickers. He gave 37 to Susan. How many stickers does David have now? $63 - 37 = ?$
- Add To: David had \$37. His grandpa gave him some money for his birthday. Now he has \$63. How much money did David's grandpa give him? $\$37 + ? = \$63$
- Compare: David has 63 stickers. Susan has 37 stickers. How many more stickers does David have than Susan? $63 - 37 = ?$
    - Even though the modeling of the two problems above is different, the equation, $63 - 37 = ?$ can represent both situations (How many more do I need to make 63?)
- Take From (Start Unknown): David had some stickers. He gave 37 to Susan. Now he has 26 stickers. How many stickers did David have before? $? - 37 = 26$

It is important to attend to the difficulty level of the problem situations in relation to the position of the unknown.

- Result Unknown, Total Unknown, and Both Addends Unknown problems are the least complex for students.
- The next level of difficulty includes Change Unknown, Addend Unknown, and Difference Unknown
- The most difficult are Start Unknown and versions of Bigger and Smaller Unknown (compare problems).

This standard focuses on developing an algebraic representation of a word problem through addition and subtraction. The intent is **NOT** to introduce traditional algorithms or rules, but to "**make meaning**" of operations.
Second graders should work on ALL problem types regardless of the level of difficulty. Mastery is expected in second grade. Students can use interactive whiteboard or document camera to demonstrate and justify their thinking.

Instructional Strategies:

Solving algebraic problems requires emphasizing the most crucial problem-solving strategy—understanding the situation.

Students now build on their work with one-step problems to solve two-step problems and model and represent their solutions with equations for all the situations shown in Table 1, Appendix.

The problems should involve sums and differences less than or equal to 100 using the numbers 0 to 100. It is important that students develop the habit of checking their answer to a problem to determine if it makes sense for the situation and the questions being asked.

Ask students to write word problems for their classmates to solve. Start by giving students the answer to a problem. Then tell students whether it is an addition or subtraction problem situation. Also let them know that the sums and differences can be less than or equal to 100 using the numbers 0 to 100.

For example, ask students to write an addition word problem for their classmates to solve which requires adding four two-digit numbers with 100 as the answer. Students then share, discuss and compare their solution strategies after they solve the problems.

Tools/Resources

See: "Pies for Sale," NCSM, Great Tasks for Mathematics K-5, (2013).
See: "Show What You Know," NCSM, Great Tasks for Mathematics K-5, (2013).

Illustrative Mathematics:
Pencil and a Sticker
NBT Saving Money 2

See: Progressions for Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten for detailed information.

Common Misconceptions:

Some students end their solution to a two-step problem after they complete the first step. They may have misunderstood the question or only focused on finding the first part of the problem.

Students need to check their work to see if their answer makes sense in terms of the problem situation.

They need many opportunities to solve a variety of two-step problems and develop the habit of reviewing their solution after they think they have finished.

Many children have misconceptions about the equal sign. Students can misunderstand the use of the equal sign even if they have proficient computational skills. The equal sign means , "is the same as" however, many primary students think that the equal sign tells you that the "answer is coming up."

Students need to see examples of number sentences with an operation to the right of the equal sign and the answer on the left, so they do not over-generalize from those limited examples.

They might also be predisposed to think of equality in terms of calculating answers rather than as a relation because it is easier for young children to carry out steps to find an answer than to identify relationships among quantities.

Students might rely on a key word or phrase in a problem to suggest an operation that will lead to an incorrect solution. They might think that the word *left* always means that subtraction must be used to find a solution.
Students need to solve problems where key words are contrary to such thinking.

- For example, the use of the word *left* does not indicate subtraction as a solution method: Debbie took the 8 stickers she no longer wanted and gave them to Anna. Now Debbie has 11 stickers *left*. How many stickers did Debbie have to begin with?

It is important that students not rely on using key words to solve problems. The goal is for students to make sense of the problem and understand what it is asking them to do, rather than search for "tricks" and/or guess at the operation needed to solve the problem.

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Add and subtract within 20.*

Standard: Grade 2.OA.2

Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction**.
- *Represent and solve problems involving addition and subtraction* and *Add and subtract within 20* in Grade 1, and to *Use place value understanding and properties of operations to add and subtract* in Grade 2

Explanation and Examples:

See standard 1.OA.6 for a list of mental strategies.

This standard mentions the word fluently when students are adding and subtracting numbers within 20. Fluency means accuracy (correct answer), efficiency (within 4-5 seconds), and flexibility (using strategies, such as making 10 or breaking apart numbers).

Mental Strategies

- Counting on
- Making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 = 4 = 14$)
- Decomposing a number leading to a ten ($13 - 3 - 1 = 10 - 1 = 9$)
- Using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$)
- Creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent $6 + 6 + 1 = 12, 12 + 1 = 13$)

Second Graders internalize facts and develop fluency by repeatedly using strategies that make sense to them.

When students are able to demonstrate fluency they are accurate, efficient, and flexible. Students must have efficient strategies in order to know sums from memory.

1. Research indicates that teachers' can best support students' memorization of sums and differences through varied experiences such as, making 10, breaking numbers apart and working on mental strategies. These strategies replace the use of repetitive timed tests in which students try to memorize operations as if there were not any relationships among the various facts. When teachers teach facts for automaticity, rather than memorization, they encourage students to THINK about the relationships among the facts. (Fostnot & Dolk, 2001)

It is no accident that the standard says "know from memory" rather than "memorize". The first describes an outcome, whereas the second might be seen as describing a method of achieving that outcome. So no, the standards are not dictating timed tests. (McCallum, 2011)

Major Supporting Additional **Depth Opportunities(DO)**

Example: 9 + 5 = ____
?

Student 1	Student 2
Counting On I started at 9 and then counted 5 more. I landed on 14.	Decomposing a Number Leading to a Ten I know that 9 and 1 is 10, so I broke 5 into 1 and 4. 9 plus 1 is 10. Then I have to add 4 more, which gets me to 14.

Which one is more efficient? Have these discussions with students so they will be flexible in their thinking.

Example: 13 – 9 = _____

=?

Student 1	Student 2
Using the Relationship between Addition and Subtraction: I know that 9 plus 4 equals 13. So, 13 minus 9 equals 4.	Creating an easier problem using compatible numbers: Instead of 13 minus 9. I added 1 to each of the numbers to make the problem 14 minus 10. I know the answer is 4. So 13 minus 9 is also 4.

This standard is strongly connected to all the standards in this domain. It focuses on students being able to fluently add and subtract numbers to 20. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Mental strategies help students **make sense** of number relationships as they are adding and subtracting within 20. The ability to calculate mentally with efficiency is very important for all students. Mental strategies may include the following:

- Counting on (works best with +1 and +2 facts)
- Making tens (9 + 7 = 10 + 6)
- Decomposing a number leading to a ten (14 – 6 = 14 – 4 – 2 = 10 – 2 = 8)
- Fact families (8 + 5 = 13 is the same as 13 - 8 = 5)
- Doubles
- Doubles plus one (7 + 8 = 7 + 7 + 1); sometimes called Near Doubles

The use of objects, diagrams, or interactive whiteboards, and various strategies will help students develop fluency.

Instructional Strategies:
An efficient strategy is one that can be done mentally and quickly. Provide many activities that will help students develop a strong understanding of number relationships, addition and subtraction so they can develop, share and use efficient strategies for mental computation.

Students gain computational fluency, using efficient and accurate methods for computing, as they come to understand the role and meaning of arithmetic operations in number systems. Efficient mental processes become automatic with use.

- Have students study how numbers are related to the anchor numbers 5 and 10, so they can apply these relationships to their strategies for knowing 5 + 4 or 8 + 3.
- Students might picture 5 + 4 on a ten-frame to mentally see 9 as the answer, or 1 less than 10.

- For remembering 8 + 7, students might think, since 8 is 2 away from 10, take 2 away from 7 to make 10+5=15 or know that 7+7= 14 and one more makes 15.
- Another example: After multiple experiences with ten-frames, when students add to 9, they mentally SEE 9, but THINK 10 and generalize that 9 + 8 is the same thing as 10 + 7. Then, apply this same thinking to 19 + 8 is the same thing as 20 + 7, SEE 19, THINK 20, and so on.

Provide activities in which students apply the commutative and associative properties to their mental strategies for sums less or equal to 20 using the numbers 0 to 20.

Provide simple word problems designed for students to invent and try a particular strategy as they solve it. Have students explain their strategies so their classmates can understand it.

Guide the discussion so the focus is on the methods that are most useful. Encourage students to try the strategies that were shared so they can eventually adopt efficient strategies that work for them.

Make posters for student-developed, mental strategies for addition and subtraction within 20. Use names for the strategies that make sense to the students and include examples of the strategies.

Present a particular strategy along with the specific addition and subtraction facts relevant to the strategy. Have students use objects and drawings to explore how these facts are alike.

Tools/Resources:
Illustrative Mathematics:
Building toward fluency
Hitting The Target Number
MD Delayed Gratification

Common Misconceptions:
Students may over-generalize and begin to think that answers to addition problems must be greater. Example: Adding 0 to any number results in a sum that is equal to that number and not greater. Provide word problems involving 0 and have students model using drawings with an empty space for 0.

Students are usually proficient when they focus on a strategy relevant to particular facts. When these facts are mixed with others, students may revert to counting as a strategy and ignore the efficient strategies they learned.
Provide a list of facts from two or more strategies and ask students to name a strategy that would work for that fact. Students should be expected to explain why they chose that strategy then show how to use it. This relates to efficiency.

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Work with equal groups of objects to gain foundation for multiplication*

Standard: Grade 2. OA.3

Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction.**
- Work with addition and subtraction equations and Use place value understanding and properties of operations to add and subtract in Grade 1, and to Use place value understanding and properties of operations to add and subtract in Grade 2.

Explanation and Examples:

This standard calls for students to apply their work with double addition facts to the concept of odd or even numbers. Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends (e.g. 10 = 5 + 5), then that number (10 in this case) is an even number.

Students should explore this concept with concrete objects (e.g., counters, place value cubes, etc.) before moving towards pictorial representations such as circles or arrays.

Example:

Is 8 an even number? Prove your answer.

Student 1	**Student 2**
I grabbed 8 counters. I paired counters up into groups of 2. Since I didn't have any counters left over, I know that 8 is an even number	I know that 4 plus 4 equals 8. So 8 is an even number.
Student 3	**Student 4**
I drew 8 boxes in a rectangle that had two columns. Since every box on the left matches a box on the right, I know that 8 is even.	I drew 8 circles. I matched one on the left with one on the right. Since they all match up I know that 8 is an even number.

Students explore odd and even numbers in a variety of ways including the following:

1. Students may investigate if a number is odd or even
2. Determining if the number of objects can be divided into two equal sets, arranged into pairs or counted by twos.
3. After the above experiences, students may discover that they only need to look at the digit in the ones place to determine if a number is odd or even since any number of tens will always split into two even groups.

Major Supporting Additional **Depth Opportunities(DO)**

Example:
Students need opportunities writing equations representing sums of two equal addends, such as: 2 + 2 = 4, 3 + 3 = 6, 5 + 5 = 10, 6 + 6 = 12, or 8 + 8 =16. This understanding will lay the foundation for multiplication and is closely connected to 2.OA.4.

The use of objects and/or interactive whiteboards will help students develop and demonstrate various strategies to determine even and odd numbers.

Instructional Strategies: (2.OA3-4)

Students need to understand that a collection of objects can be one thing or one group and that a group contains a given number of objects.

- Investigate separating no more than 20 objects into two equal groups.
- Find the numbers that will have some objects remaining and no objects remaining after separating the collections into two equal groups.
- Odd numbers will have some objects remaining while even numbers will not. For an even number of objects in a collection, show the total as the sum of equal addends (repeated addition).

Tools/Resources:

Illustrative Mathematics:
Red and Blue Tiles
Buttons odd and even
Counting Dots in Arrays

Common Misconceptions:

Students will look at the number of digits to determine if the number is odd or even instead of the quantity itself. Example: 53 is an even number because it has 2 digits. This is a misconception.

Students will determine whether a number is odd or even by the first digit in the number instead of the digit in the ones place.

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Work with equal groups to gain foundations for multiplication*

Standard: Grade 2.OA.4

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2.OA.3

Explanation and Examples:

This standard calls for students to use rectangular arrays to work with repeated addition. This is a building block for multiplication in 3rd Grade. Students should explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings.

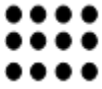
Based on the commutative property of addition, students can add either the rows or the columns and still arrive at the same solution.

Example

Find the total number of objects below. (Students use concrete objects to show, solve and explain)

Student 1	Student 2
I see 3 counters in each column and there are 4 columns. So I added 3 + 3 + 3 + 3. That equals 12.	I see 4 counters in each row and there are 3 rows. So I added 4+4+4. That equals 12.

Students arrange any set of objects into a rectangular array. Objects can be cubes, buttons, counters, etc. Objects do not have to be square to make an array. *Geoboards* can also be used to demonstrate rectangular arrays. Students then write equations that represent the total as the sum of equal addends as shown below



4 + 4 + 4 + 4 =

5 + 5 + 5 + 5 = 20

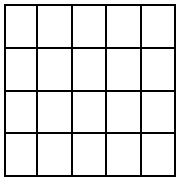
Interactive whiteboards and document cameras may be used to help students visualize and create arrays.

Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies:

A rectangular array is an arrangement of objects in horizontal rows and vertical columns. Arrays can be made out of any number of objects that can be put into rows and columns.

- All rows contain the same number of items and all columns contain an equal number of items.
- Have students use objects to build all the arrays possible with no more than 25 objects. Their arrays should have up to 5 rows and up to 5 columns.
- Ask students to draw the arrays on grid paper and write two different equations under the arrays: one showing the total as a sum by rows and the other showing the total as a sum by columns.
- Both equations will show the total as a sum of equal addends.



The equation by rows: 20 = 5 + 5 + 5 +5

The equation by columns: 20 = 4 + 4 + 4 + 4 + 4

Build on knowledge of composing and decomposing numbers to investigate arrays with up to 5 rows and up to 5 columns in different orientations.

For example, form an array with 3 rows and 4 objects in each row. Represent the total number of objects with equations showing a sum of equal addends two different ways: by rows, 12 = 4 + 4 + 4; by columns, 12 = 3 + 3 + 3 + 3.

Show that by rotating the array 90° to form 4 rows with 3 objects in each row. Write two different equations to represent 12 as a sum of equal addends: by rows, 12 = 3 + 3 + 3 + 3; by columns, 12 = 4 + 4 + 4.

Have students discuss this statement and explain their reasoning: The two arrays are different and yet the same.

Ask students to think of a full ten-frame showing 10 circles as an array. One view of the ten-frame is 5 rows with 2 circles in each row.

Students count by rows to 10 and write the equation 10 = 2 + 2 + 2 + 2 + 2. Then students put two full ten-frames together end-to-end so they form 10 rows of 2 circles or (10 columns of 2 circles).

They use this larger array to count by 2s up to 20 and write an equation that shows 20 equal to the sum of ten 2s.

Tools/Resources:
Illustrative Mathematics:
Counting Dots in Arrays

See Learning Progressions for Operations & Algebraic Thinking for detailed information

Common Misconceptions:
Students may not be consistent in how they describe arrays. They may not remember the convention that was discussed in class. They cannot justify that the commutative property applies.

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Understand place value.*

Standard: Grade 2.NBT.1

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens—called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- Second Grade Critical Area of Focus #1, **Extending understanding of base-ten notation**.
- *Extend the counting sequence* and *Understand place value* in Grade 1
- *Work with equal groups of objects to gain foundations for multiplication* in Grade 2
- *Use place value understanding and properties of operations to perform multi-digit arithmetic* in Grade 3.

Explanation and Examples:

This standard calls for students to work on decomposing numbers by place value. Students should have ample experiences with concrete materials and pictorial representations examining that numbers all numbers between 100 and 999 can be decomposed into hundreds, tens, and ones and then into several different combinations.

Example:

285 can be shown as 2 hundreds, 8 tens, and 5 ones but it is also correct to show as 28 tens and 5 ones OR 1 hundred, 18 tens, and 5 ones and so on.

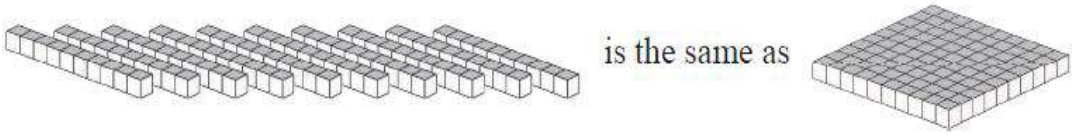
Interpret the value of a digit (1-9 and 0) in a multi-digit numeral by its position within the number with models, words and numerals.

Use 10 as a benchmark number to compose and decompose when adding and subtracting whole numbers.

2.NBT.1a calls for students to extend their work from 1st Grade by exploring a hundred as a unit (or bundle) of ten tens.

2.NBT.1b builds on the work of **2.NBT.2a.** Students should explore the idea that numbers such as 100, 200, 300, etc., are groups of hundreds that have no tens or ones. Students can represent this with place value (base 10) blocks.

Major Supporting Additional **Depth Opportunities(DO)**



6 hundreds are the same as 600

Understanding that 10 ones make one ten and that 10 tens make one hundred is fundamental to students' mathematical development.

Students need multiple opportunities counting and "bundling" groups of tens in first grade. In second grade, students build on their understanding by making bundles of 100s with or without leftovers using base ten blocks, cubes in towers of 10, ten frames, etc. This emphasis on bundling hundreds will support students' discovery of place value patterns.

As students are representing the various amounts, it is important that emphasis is placed on the language associated with the quantity.

For example, 243 **can be expressed in multiple ways** such as 2 groups of hundred, 4 groups of ten and 3 ones, as well as 24 tens and 3 ones.

When students read numbers, they should read in standard form as well as using place value concepts. For example, 243 should be read as "two hundred forty-three" as well as two hundreds, 4 tens, 3 ones.

A document camera or interactive whiteboard can also be used to demonstrate "bundling" of objects. This gives students the opportunity to communicate their thinking.

Instructional Strategies: (2.NBT1-4)

The understanding that 100 is 10 tens or 100 ones is critical to the understanding of place value. Using proportional models like base-ten blocks and bundles of tens along with numerals on place-value mats provides connections between physical and symbolic representations of a number. These models can be used to compare two numbers and identify the value of their digits.

Model three-digit numbers using base-ten blocks in multiple ways. For example, 236 can be 236 ones, OR 23 tens and 6 ones, OR 2 hundreds, 3 tens and 6 ones, OR 20 tens and 36 ones. Use activities and games that have students match different representations of the same number.

Provide games and other situations that allow students to practice skip-counting. Students can use nickels, dimes and dollar bills to skip count by 5, 10 and 100. Pictures of the coins and bills can be attached to models familiar to students: a nickel on a five-frame with 5 dots or pennies and a dime on a ten-frame with 10 dots or pennies.

On a number line, have students use a clothespin or marker to identify the number that is ten more than a given number or five more than a given number.

Have students create and compare all the three-digit numbers that can be made using numbers from 0 to 9. For instance, using the numbers 1, 3, and 9, students will write the numbers 139, 193, 319, 391, 913 and 931.
When students compare the numerals in the hundreds place, they should conclude that the two numbers with 9 hundreds would be greater than the numbers showing 1 hundred or 3 hundreds. When two numbers have the same digit in the hundreds place, students need to compare their digits in the tens place to determine which number is larger.

Tools/Resources:
Illustrative Mathematics:
Regrouping
Three composing/decomposing problems
Bundling and Unbundling
Boxes and Cartons of Pencils
Largest Number Game
Ten $10s make $100
Counting Stamps
One, Ten, and One Hundred More and Less
Making 124
Looking at Numbers Every Which Way
Party Favors

See: Progression for Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten for detailed information.

Common Misconceptions: (2.NBT.1-4)
Some students may not move beyond thinking of the number 358 as 300 ones plus 50 ones plus 8 ones to the concept of 8 singles, 5 bundles of 10 singles or tens, and 3 bundles of 10 tens or hundreds. Use base-ten blocks to model the collecting of 10 ones (singles) to make a ten (a rod) or 10 tens to make a hundred (a flat). It is important that students connect a group of 10 ones with the word *ten* and a group of 10 tens with the word *hundred.*

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Understand Place value.*

Standard: Grade 2.NBT.2

Count within 1000; skip-count by 5s, 10s, and 100s.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2. NBT.1

Explanation and Examples:

The standard calls for students to count within 1,000. This means that students are expected to "count on" from any number and say the next few numbers that come afterwards.

Understand that counting by 2s, 5s and 10s is counting groups of items by that amount.

Example:

What are the next 3 numbers after 498? *499, 500, 501*.

When you count back from 201, what are the first 3 numbers that you say? *200, 199, 198*.

This standard also introduces skip counting by 5s and 100s. Students are introduced to skip counting by 10s in First Grade.

Students should explore the patterns of numbers when they skip count. When students skip count by 5s, the ones digit alternates between 5 and 0. When students skip count by 100s, the hundreds digit is the only digit that changes, and it increases by one number.

Students need many opportunities counting, up to 1000, from different starting points (Example: Skip count by 3s starting at 10). They should also have many experiences skip counting by 5s, 10s, and 100s to develop the concept of place value.

Examples:

- The use of the 100s chart may be helpful for students to identify the counting patterns.
- The use of money (nickels, dimes, dollars) or base ten blocks may be helpful visual cues.
- The use of an interactive whiteboard may also be used to develop counting skills.

The ultimate goal for second graders is to be able to count in multiple ways with no visual support.

Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies: **See Grade 2. NBT.1**

Tools/Resources:
Illustrative Mathematics:
Saving Money 2

Common Misconceptions: See Grade 2. NBT.1

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Understand place value.*

Standard: Grade 2.NBT.3

Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2.NBT.1

Explanation and Examples:

This standard calls for students to read, write and represent a number of objects with a written numeral (number form or standard form). These representations can include place value (base 10) blocks, pictorial representations or other concrete materials. Remember that when reading and writing whole numbers, the word "*and*" should **not** be used between any of the whole-number words – "*and*" represents the decimal point.

Example:
235 is written and spoken as *two hundred thirty-five*.

Students need many opportunities reading and writing numerals in multiple ways.

Examples:

• Base-ten numerals	637	(standard form)
• Number names	six hundred thirty seven	(written form)
• Expanded form	600 + 30 + 7	(expanded notation)
Short word form can also be used	*6 hundreds + 3 tens + 7 ones*	

When students say the expanded form, it may sound like this: "6 hundreds plus 3 tens plus 7 ones" OR 600 plus 30 plus 7."

Instructional Strategies: See Grade 2.NBT.1

Tools/Resources:

Illustrative Mathematics:
Looking at Numbers Every Which Way

Common Misconceptions: See Grade 2.NBT.1

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Understand place value.*

Standard: Grade 2.NBT.4

Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2.NBT.1

Explanation and Examples:

This standard builds on the work of **2.NBT.1** and **2.NBT.3** by having students compare two numbers by examining the amount of hundreds, tens and ones in each number.

Students are introduced to the symbols greater than (>), less than (<) and equal to (=) in First Grade, and use them in Second Grade with numbers within 1,000.

Students should have ample experiences communicating their comparisons **in words** before using only symbols in this standard.

Example: 452 _____ 455

Student 1	Student 2
452 has 4 hundreds 5 tens and 2 ones. 455 has 4 hundreds 5 tens and 5 ones. They have the same number of hundreds and the same number of tens, but 455 has 5 ones and 452 only has 2 ones,452 is less than 455. 452 < 455	452 is less than 455. I know this because when I count up I say 452 before I say 455.

Students may use models, number lines, base ten blocks, interactive whiteboards, document cameras, written words, and/or spoken words that represent two three-digit numbers.

To compare, students apply their understanding of place value. They first attend to the numeral in the hundreds place, then the numeral in tens place, then, if necessary, to the numeral in the ones place.

Comparative language includes but is not limited to: more than, less than, greater than, most, greatest, least, same as, equal to and not equal to. Students use the appropriate symbols to record the comparisons.

Instructional Strategies: **See Grade 2.NBT.1**

Major Supporting Additional **Depth Opportunities(DO)**

Tools/Resources:

Illustrative Mathematics:

Ordering 3-digit numbers

Comparisons 2

Comparisons 1

Digits 2-5-7

Number Line Comparisons

Using Pictures to Explain Number Comparisons

Common Misconceptions: See Grade 2.NBT.1

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Use place value understanding and properties to add and subtract.*

Standard: Grade 2. NBT.5

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: (2.NBT.5-9)

This cluster is connected to:

- Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction**.
- *Understand and apply properties of operations and the relationship between addition and subtraction* and *Add and subtract within 20 and Use place value understanding and properties of operations to add and subtract* in Grade 1
- *Add and subtract within 20* in Grade 2
- *Use place value understanding and properties of operations to perform multi-digit arithmetic* in Grade 3.

Explanation and Examples:

This standard mentions the word fluently when students are adding and subtracting numbers within 100. Fluency means accuracy (correct answer), efficiency (reasonable steps and time in computing), and flexibility (using strategies such as making 10 or breaking numbers apart).

Example: 67 + 25 =

Place Value Strategy	**Counting On and Decomposing a Number Leading to a Ten**	**Associative Property:**
I broke both 67 and 25 into tens and ones. 6 tens plus 2 tens equals 8 tens. Then I added the ones. 7 ones plus 5 ones equals 12 ones. I then combined my tens and ones. 8 tens plus 12 ones equals 92.	I wanted to start with 67 and then break 25 apart. I started with 67 and counted on to my next ten. 67 plus 3 gets me to 70. I then added 2 more to get to 72. I then added my 20 and got to 92.	I broke 67 and 25 into tens and ones so I had to add 60+7+20+5. I added 60 and 20 first to get 80. Then I added 7 to get 87. Then I added 5 more. My answer is 92.

Example: 63 - 32 =

Relationship between Addition and Subtraction
I broke apart both 63 and 32 into tens and ones. I know that 2 plus 1 equals 3, so I have 1 left in the ones place. I know that 3 plus 3 equals 6, so I have a 3 in my tens place. My answer has a 1 in the ones place and 3 in the tens place, so my answer is 31.

Major Supporting Additional **Depth Opportunities(DO)**

Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Students should have experiences solving problems written both horizontally and vertically. They need to communicate their thinking and be able to justify their strategies both verbally and with paper and pencil.

Addition strategies based on place value for 48 + 37 may include:

- Adding by place value: 40 + 30 = 70 and 8 + 7 = 15 and 70 + 15 = 85.
- Incremental adding (breaking one number into tens and ones); 48 + 10 = 58, 58 + 10 = 68, 68 + 10 = 78, 78 + 7 = 85
- Compensation (making a friendly number): 48 + 2 = 50, 37 – 2 = 35, 50 + 35 = 85

Subtraction strategies based on place value for 81 - 37 may include:

- Adding up (from smaller number to larger number): 37 + 3 = 40, 40 + 40 = 80, 80 + 1 = 81, and 3 + 40 + 1 = 44.
- Incremental subtracting: 81 -10 = 71, 71 – 10 = 61, 61 – 10 = 51, 51 – 7 = 44
- Subtracting by place value: 81 – 30 = 51, 51 – 7 = 44

Properties that students should know and use are:

- Commutative property of addition (Example: 3 + 5 = 5 + 3)
- Associative property of addition (Example: (2 + 7) + 3 = 2 + (7+3))
- Identity property of 0 (Example: 8 + 0 = 8)

Students in second grade need to communicate their understanding of why some properties work for some operations and not for others.

- **Commutative Property:** In first grade, students investigated whether the commutative property works with subtraction. The intent was for students to recognize that taking 5 from 8 is not the same as taking 8 from 5. Students should also understand that they will be working with numbers in later grades that will allow them to subtract greater numbers from smaller numbers. This exploration of the commutative property continues in second grade.

Instructional Strategies:

Provide many activities that will help student develop a strong understanding of number relationships, addition and subtraction so they can develop, share and use efficient strategies for mental computation. An efficient strategy is one that can be done mentally and quickly. Students gain computational fluency, using efficient and accurate methods for computing as they come to understand the role and the meaning of arithmetic operations in the number system.

Students need to build on their flexible strategies for adding within 100 in Grade 1 to fluently add and subtract within 100, add up to four two-digit numbers and find the sums and differences less than or equal to 1000 using numbers 0 to 1000.

Initially students apply base-ten concepts and use direct modeling with physical objects or drawings to find different ways to solve problems. They move to inventing strategies that do not involve physical material or counting by ones to solve problems. Student invented strategies likely will be based on place value concepts, the commutative and associative properties and the relationship between addition and subtraction. These strategies should be done mentally or with written record for support.

Major Supporting Additional **Depth Opportunities(DO)**

It is vital that student-invented strategies be shared, explored, recorded and tried by others. Recording expressions and equations in the strategies horizontally encourages student to think about the numbers and the quantities they represent instead of the digits.

Not every student will invent strategies, but all student can and will try strategies they have seen that makes sense to them. Different strategies will be chosen by different students.

Students will decompose and compose tens and hundreds when they develop their own strategies for solving problems where regrouping is necessary. They might use the make-ten strategy (37 + 8 = 40 + 5) add 3 to 37 then 5, or (62 + 9 = 60 + 7) take off 2 and get 60 then add 7 more) because no ones are exchanged for a ten or a ten for ones.

Have students analyze problems before they solve them. Present a variety of subtraction problems within 1000. Ask students to identify the problems requiring them to decompose the tens or hundreds to find a solution and explain their reasoning.

Tools/Resources:

Illustrative Mathematics:

Jamir's Penny Jar

Saving Money 2

Saving Money 1

See: "Show What You Know," NCSM, Great Tasks for Mathematics K-5, (2013).

Common Misconceptions: (2.NBT.5-9)

Students may think that the 4 in 46 represents 4, not 40 or 4 tens. Students need many experiences representing two- and three-digit numbers with manipulatives that group (base ten blocks) and those that do NOT group, such as counters, etc.

When adding two-digit numbers, some students might start with the digits in the ones place and record the entire sum. Then they add the digits in the tens place and record this sum. Assess students' understanding of *a ten* and provide more experiences modeling addition with grouped and pre-grouped base-ten materials as mentioned above.

When subtracting two-digit numbers, students might start with the digits in the ones place and subtract the smaller digit from the greater digit. Then they move to the tens and the hundreds places and subtract the smaller digits from the greater digits. Assess students' understanding of *a ten* and provide more experiences modeling subtraction with grouped and pre-grouped base-ten materials.

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Use place value understanding and properties of operations to add and subtract.*

Standard: Grade 2.NBT.6

Add up to four two-digit numbers using strategies based on place value and properties of operations.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2.NBT.5

Explanation and Examples:

This standard calls for students to add a string of two-digit numbers (up to four numbers) by applying place value strategies and properties of operations.

Example: 43 + 34 + 57 + 24 = _____

Student 1 - Associative Property	**Student 2 - Place Value Strategies**
I saw the 43 and 57 and added them first, since I know 3 plus 7 equals 10. When I added them 100 was my answer. Then I added 34 and had 134. Then I added 24 and had 158.	I broke up all of the numbers into tens and ones. First I added the tens. 40 + 30 + 50 + 20 = 140. Then I added the ones. 3 + 4 + 7 + 4 = 18. Then I combined the tens and ones and had 158 as my answer.
Student 3 - Place Value Strategies and Associative Property	**Student 4**
I broke up all the numbers into tens and ones. First I added up the tens. 40 + 30 + 50 + 20. I changed the order of the numbers to make adding easier.	I added up the ones. 3 + 4 + 7 + 4. I changed the order of the numbers to make adding easier. I know that 3 plus 7 equals 10 and 4 plus 4 equals 8. 10 plus 8 equals 18. I then combined my tens and my ones. 140 plus 18 equals 158.

Students demonstrate addition strategies with up to four two-digit numbers either with or without regrouping. Problems may be written in a story problem format to help develop a stronger understanding of larger numbers and their values. Interactive whiteboards and document cameras may also be used to model and justify student thinking.

Instructional Strategies: See Grade 2.NBT.5

Tools/Resources:

Illustrative Mathematics:

Toll Bridge Puzzle

Common Misconceptions: See Grade 5 NBT.5

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Use place value understanding and properties of operations to add and subtract.*

Standard: Grade 2. NBT.7

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2.NBT.5

Explanation and Examples:

This standard builds on the work from 2.NBT.5 by increasing the size of numbers (two 3-digit numbers). Students should have ample experiences to use concrete materials (place value blocks) and pictorial representations to support their work.

This standard also references composing and decomposing a ten. This work should include strategies such as making a 10, making a 100, breaking apart a 10, or creating an easier problem. While the standard (traditional) algorithm could be used here, students' experiences should extend beyond only working with the algorithm.

There is a strong connection between this standard and place value understanding with addition and subtraction of smaller numbers. Students may use concrete models or drawings to support their addition or subtraction of larger numbers.

Strategies are similar to those stated in 2.NBT.5, as students extend their learning to include greater place values moving from tens to hundreds to thousands. Interactive whiteboards and document cameras may also be used to model and justify student thinking.

Students use number lines, base ten blocks, etc. to show, solve and explain reasoning. Explanation of thinking is a critical component of this standard.

Major Supporting Additional **Depth Opportunities(DO)**

Example: 354 + 287 =

Student 1	Student 2	Student 3
Uses a number line. "I started at 354 and jumped 200. I landed on 554. I then made 8 jumps of 10 and landed on 634. I then jumped 7 and landed on 641."	Uses base ten blocks & mat. "I broke all of the numbers up by place using a place value chart. I first added the ones (4+7), then the tens (50+80) and then the hundreds200=500) I then combined my answers: 500+130=630. 630+11=641".	Uses place value blocks. "I made a pile of 354. I then added 287. That gave me 5 hundreds, 13 tens and 11 ones. I noticed that I could trade some pieces. I had 11 ones, and traded 10 ones for a ten. I then had 14 tens, so I traded 10 tens for a hundred and ended up with 6 hundreds, 4 tens and 1 one

Instructional Strategies: **See Grade 2.NBT.5**

Tools/Resources:

Illustrative Mathematics:

How Many Days Until Summer Vacation?

Many Ways to do Addition 2

See: Progression for Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten detailed information.

See: "Creating Story Problems ", Georgia Department of Education. In this lesson, students will add and subtract numbers less than 100 and understand the relationship between addition and subtraction. The activity applies reading/listening to story problems. Students write and solve problems involving a variety of situations, choosing strategies including- part-part- whole, comparing, grouping, doubling, counting on and counting back situations. Students will use drawings, equations, and written responses to solve single story problems

Common Misconceptions: See Grade 2.NBT.5

Domain: Number and operations in Base Ten (NBT)

Cluster: *Use place value understanding and properties of operations to add and subtract.*

Standard: Grade 2.NBT.8

Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100 – 900.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2. NBT.5

Explanation and Examples:

This standards calls for students to mentally add and subtract multiples of 10 or 100 to any number between 100 and 900. When computing sums of 3-digit numbers, students might use strategies based on flexible combination of Level 3 *composition and decomposition* and Level 2 *counting-on strategies* when finding the value of an expression such as 148 + 473. (See Table 6 in Appendix) Example: they might say "100 and 400 is 500. And 70 and 30 is another hundred, so 600. Then, 8, 9, 10, 11…..and the other 10 is 21. So, 621". Keeping track of what is being added is easier using a written form of such reasoning and makes it easier to discuss. There are two kinds of decompositions in this strategy. Both addends are decomposed into 100s, 10s, and ones, and the first addend decomposed successively into the part already added and the part still to add.

Students should have ample experiences developing proficiency with mental computation. Mentally adding and subtracting 10 or 100 to a given number understanding that they are only changing the tens place (multiples of ten) or the digit in the hundreds place (multiples of 100). Working with place value blocks before moving to mental computation will be beneficial for most students.

Problems that require students to move from 10's to 100's should be included.

Example: 273 + 60 = 333

Explorations should also include looking for relevant patterns.

Mental math strategies may include:

- Counting on; 300, 400, 500, etc.
- Counting back; 550, 450, 350, etc.

Major Supporting Additional **Depth Opportunities(DO)**

Examples:

- 100 more than 653 is _____(753)
- 10 less than 87 is _____(77)
- "Start at 248. Count up by 10s until I tell you to stop."

An interactive whiteboard or document camera may be used to help students develop these mental math skills.

Instructional Strategies: **See Grade 2.NBT.5**

See: "Piggy Banks", NCSM, Great Tasks for Mathematics K-5, (2013).

Common Misconceptions: See Grade 2.NBT.5

Domain: Number and Operations in Base Ten (NBT)

***Cluster**: Use place value understanding and properties of operations to add and subtract.*

Standard: Grade 2.NBT.9

Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2.NBT.5

Explanation and Examples:

This standard calls for the student to explain using concrete objects, pictures and words (oral or written) **why** addition or subtraction strategies work. The expectation is that students apply their knowledge of place value and the properties of operations in their explanation. This is where it is critical that they see the connection between standards. Understanding place value and then being able to describe how it works when computing, along with the properties of operations, is expected. Students don't need to name the properties but they need to know that they can use them and how to use them when computing.

Students should have the opportunity to solve problems and then explain why their strategies work – using place value language and/or the use of the properties of operations.

Example:

There are 36 birds in the park. 25 more birds arrive. How many birds are there? Solve the problem and show your work.

Student 1	**Student 2**
I broke 36 and 25 into tens and ones and then added them. 30 + 6 + 20 + 5. I can change the order of my numbers, so I added 30+20 and got 50. Then I added on 6 to get 56. Then I added 5 to get 61. This strategy works because I broke all the numbers up by their place value.	I used place value blocks and made a pile of 36. Then I added 25. I had 5 tens and 11 ones. I had to trade 10 ones for a 10. Then I had 6 tens and 1 one. That makes 61. This strategy works because I added up the tens and then added up the ones and traded if I had more than 10 ones.

Students could also have experiences examining strategies and explaining why they work. Also include incorrect examples for students to examine. Operations embedded within meaningful context promote development of reasoning and justification.

Major Supporting Additional **Depth Opportunities(DO)**

Example:
One of your classmates solved the problem 56 - 34 = __ by writing —I know that I need to add 2 to the number 4 to get 6. I also know that I need to add 20 to 30 to get 20 to get to 50. So, the answer is 22.‖ Is their strategy correct? Explain why or why not?

Example:
One of your classmates solved the problem 25 + 35 by adding 20 + 30 + 5 + 5. Is their strategy correct? Explain why or why not?

Example:
Mason read 473 pages in June. He read 227 pages in July. How many pages did Mason read altogether?

Karla's explanation:	**Debbie's explanation:**	**Becky's explanation:**
473 + 227 = _____. I added the ones together (3 + 7) and got 10. Then I added the tens together (70 + 20) and got 90. I knew that 400 + 200 was 600. So I added 10 + 90 for 100 and added 100 + 600 and found out that Mason had read 700 pages altogether.	473 + 227 = _____. I started by adding 200 to 473 and got 673. Then I added 20 to 673 and I got 693 and finally I added 7 to 693 and I knew that Mason had read 700 pages altogether.	I used base ten blocks on a base ten mat to help me solve this problem. I added 3 ones (units) plus 7 ones and got 10 ones which made one ten. I moved the 1 ten to the tens place. I then added 7 tens rods plus 2 tens rods plus 1 tens rod and got 10 tens or 100. I moved the 1 hundred to the hundreds place. Then I added 4 hundreds plus 2 hundreds plus 1 hundred and got 7 hundreds or 700. So Mason read 700 books.

Students should be able to connect different representations and explain the connections. Representations can include numbers, words (including mathematical language), pictures, number lines, and/or physical objects. Students should be able to use any/all of these representations as needed.

An interactive whiteboard or document camera can be used to help students develop and explain their thinking.

Instructional Strategies: **See Grade 2.NBT.5**

Common Misconceptions: See Grade 2.NBT.5

Domain: Measurement and Data (MD)

Cluster: *Measure and estimate lengths in standard units.*

Standard: Grade 2.MD.1

Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes**.**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

This cluster is connected to:

- Second Grade Critical Area of Focus #3, **Using standard units of measure**.
- *Measure lengths indirectly and by iterating length unit*s in Grade 1, and to *Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures* in Grade 3.

Explanation and Examples:

This standard calls for students to measure the length of objects in both customary (inches and feet) and metric (centimeters and meters). Students should have ample experiences choosing objects, identifying the appropriate tool and unit, and then measuring the object. The teacher should allow students to determine which tools and units to use.

Foundational understandings to help with measure concepts:

- Understand that larger units can be subdivided into equivalent smaller units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration).
- Understand the relationship between the size of a unit and the number of units needed (compensatory principal).
- Understand measuring two-dimensional space (area) using non-standard units.

Students in second grade will build upon what they learned in first grade from measuring length with non-standard units to the new skill of measuring length in metric and U.S. Customary with standard units of measure.

They should have many experiences measuring the length of objects with rulers, yardsticks, meter sticks, and tape measures.

They will need to be taught how to actually use a ruler appropriately to measure the length of an object especially as to where to begin the measuring It is important to help students locate the starting point on the measuring instrument, especially when some have a protected edge. (This is the process of justification of object and the instrument). The use of how to use rulers needs to be targeted after students understand the attributes that are to be measured. John Van de Walle has noted that if we don't make sure students understand the attributes sufficiently then our lessons focus on using the instrument and the understanding of "what" is being measured is missed and leads to misconceptions.

Major Supporting Additional **Depth Opportunities(DO)**

Ask students questions such as: “Do you start at the end of the ruler or at the zero?” helps them focus on where to start on the instrument. They ask them: “Why do we have to start at the zero?” and “Are we looking at the spaces or the tic marks on the rulers?”

Instructional Strategies: **(2.MD.1-4)**

Second graders are transitioning from measuring lengths with informal or nonstandard units to measuring with these standard units: inches, feet, centimeters, and meters. The measure of length is a count of how many units are needed to match the length of the object or distance being measured. Emphasize that the space is what is being measured, not the tic marks on the ruler.

Students have to understand what a length unit is and how it is used to find a measurement. They need many experiences measuring lengths with appropriate tools so they can become very familiar with the standard units and estimate lengths.

Use language that reflects the approximate nature of measurement, such as the length of the room is about 26 feet.

Insist that students always estimate lengths before they measure. Estimation helps them focus on the attribute to be measured, the length units, and the process. After they find measurements, have students discuss the estimates, their procedures for finding the measurements and the differences between their estimates and the measurements.

If asking student to estimate and measure more than one object, the sequence is –estimate, measure, estimate, measure…. Using this sequence helps the student refine their ability to estimate.

Rulers that have only one system (either customary or metric) work most effectively with student beginning this stage of learning to measure.

Resources/Tools:

Illustrative Mathematics:
How Big is a Foot?

For detailed information--See Learning Progressions on Measurement
Learning Progression on Data:
Centimeter rulers and tapes
Inch rulers and tapes
Yardsticks and Meter sticks

Common Misconceptions:
When some students see standard rulers with numbers on the markings, they believe that the numbers are counting the marks instead of the units or spaces between the marks.

Have students use informal or standard length units to make their own rulers by marking each whole unit with a number in the middle. They will see that the ruler is a representation of a row of units and focus on the spaces.

Some students might think that they can only measure lengths with a ruler starting at the left edge. Provide situations where the ruler does not start at zero. For example, a ruler is broken and the first inch number that can be seen is 2. If a pencil is measured and it is 9 inches on this ruler, the students must subtract 2 inches from the 9 inches to adjust for where the measurement started. Some student become confused when the ruler they are using have both customary and metric measures on it. By covering on scale with masking tape the student becomes less confused.

Domain: Measurement and Data (MD)

***Cluster**: Measure and estimate lengths in standard units.*

Standard: Grade 2.MD.2
Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 2.MD.1

Explanation and Examples:
This standard calls for students to measure an object using two units of different lengths. Concentrate on the "spaces" for the units and not the marks on the rulers.

Example:
A student measures the length of their desk and finds that it is 3 feet and 36 inches.

Students should explore the idea that the number of units for length of the desk is greater in inches than in feet. This concept is referred to as the compensatory principle.

Students need multiple opportunities to measure using different units of measure. They should not be limited to measuring to one standard unit. Students should have access to tools, both U.S. Customary and metric. The more students work with a specific unit of measure, the better they become at choosing the appropriate tool when measuring.

Students measure the length of the same object using different tools (ruler with inches, ruler with centimeters, a yardstick, or meter stick). This will help students learn which tool is more appropriate for measuring a given object.

They describe the relationship between the size of the measurement unit and the number of units needed to measure something. For instance, a student might say, "The longer the unit, the fewer I need."

Multiple opportunities to explore provide the foundation for relating metric units to customary units, as well as relating within customary (inches to feet to yards) and within metric (centimeters to meters).

Instructional Strategies: **See Grade 2.MD.1**

Common Misconceptions: See Grade 2.MD.1

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Measurement and Data (MD)

Cluster: *Measure and estimate lengths in standard units.*

Standard: Grade 2.MD.3

Estimate lengths using units of inches, feet, centimeters, and meters.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections: See Grade 2.MD.1

Explanation and Examples:

This standard calls for students to estimate the lengths of objects using inches, feet, centimeters, and meters. Students should make estimates after seeing a benchmark unit, such as the length of one inch, before making their estimate.

Example:

Estimation helps develop familiarity with the specific unit of measure being used. To measure the length of a shoe, knowledge of an inch or a centimeter is important so that one can approximate the length in inches or centimeters. Setting up personal benchmarks (such as the width of their pinky finger is a cm can be beneficial. (See list below) Students should begin practicing estimation with items which are familiar to them (length of desk, pencil, favorite book, etc.).

Some useful benchmarks for measurement are: (these are for adults so they will need to be adjusted for 2nd graders)

- First joint to the tip of a thumb is about an inch
- Length from your elbow to your wrist is about a foot
- If your arm is held out perpendicular to your body, the length from your nose to the tip of your fingers is about a yard

Instructional Strategies: See Grade 2.MD.1

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Measurement and Data (MD)

Cluster: *Measure and estimate lengths in standard units.*

Standard: Grade 2.MD.4

Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections: See Grade 2.MD.1

Explanation and Examples:

This standard calls for students to determine the difference in length between two objects. Students should choose objects, identify appropriate tools and units, measure both objects, and then determine the differences in lengths using the same unit of measure

Second graders should be familiar enough with inches, feet, yards, centimeters, and meters to be able to compare the differences in lengths of two objects. They can make direct comparisons by measuring the difference in length between two objects by laying them side by side and selecting an appropriate standard length unit of measure.

Students should use comparative phrases such as "It is longer by 2 inches" or "It is shorter by 5 centimeters" to describe the difference between two objects.

It is important that students have multiple opportunities to work with actual objects in the process of measuring.

Instructional Strategies: **See Grade 2.MD.1**

Common Misconceptions: See Grade 2.MD.1

Domain: Measurement and Data (MD)

***Cluster**: Relate addition and subtraction to length.*

Standard: Grade 2.MD.5

Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: (2.MD.5-6)

This cluster is connected to:

- Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction**.
- *Use place value understanding and properties of operations to add and subtract* in Grade 1,
- *Represent and solve problems involving addition and subtraction* in Grade 2,
- *Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures* in Grade 3.

Explanation and Examples:

This standard applies the concept of length to solve addition and subtraction word problems with numbers within 100. Students must use the same unit in these problems and discuss why it doesn't work to use different units of measure.

Example:

In P.E. class Kate jumped 14 inches. Mary jumped 23 inches. How much farther did Mary jump than Kate? Write an equation and then solve the problem.

Student 1	**Student 2**
My equation is 14 + __ = 23 since I am trying to find out the difference between Kate and Mary's jump. I used **place value blocks** and counted out 14. I then added blocks until I got to 23. I needed to add 9 blocks. Mary jumped 9 more inches than Kate.	My equation is 23 - 14 = __. I drew a **number line**. I started at 23. I moved back to 14 and counted how far I moved (the units). I moved back 9 spots. Mary jumped 9 more inches than Kate.

Students need experience working with addition and subtraction to solve word problems (make connections to all the subtypes within Table 1 (See Appendix page 67) which include measures of length. It is important that word problems stay within the same unit of measure.

Some representations students can use include drawings, number lines, rulers, pictures, and/or physical objects.
An interactive whiteboard or document camera may be used to help students develop and demonstrate their thinking.

Major Supporting Additional **Depth Opportunities(DO)**

As students begin to work measurement problems, remember to consider the different types of equations that can be used to create problems.
Equations include:

- $10 + 15 = c$
- $c - 20 = 5$
- $c - 10 = 25$
- $20 + b = 35$
- $15 + a = 35$
- $35 = a + 15$
- $35 = 20 + b$

Example:

- A word problem for $5 - n = 2$ could be: Mary is making a dress. She has 5 yards of fabric. She uses some of the fabric and has 2 yards left. How many yards did Mary use?

There is a strong connection between this standard and demonstrating fluency of addition and subtraction facts. Addition facts through 10 + 10 and the related subtraction facts should be included.

Instructional Strategies: (2.MD.5-6)

Connect the whole-number units on rulers, yardsticks, meter sticks and measuring tapes to number lines showing whole-number units starting at 0. Use these measuring tools to model different representations for whole-number sums and differences less than or equal to 100 using the numbers 0 to 100.

Use the meter stick to view units of ten (10 cm) and hundred (100 cm), and to skip count by 5s and 10s.

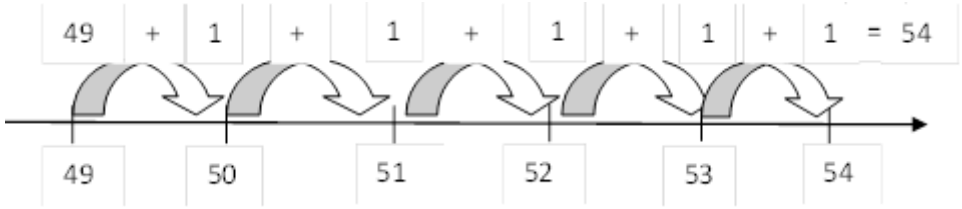
Provide one- and two-step word problems that include different lengths measurement made with the same unit (inches, feet, centimeters, or meters). Students add and subtract within 100 to solve problems for these situations:

- adding to, taking from,
- putting together,
- taking apart,
- comparing,

with **unknowns in all positions**.

Students use drawings and write equations with a symbol for the unknown to solve the problems.

- Have students represent their addition and subtraction within 100 on a number line. They can use notebook or grid paper to make their own number lines.
- First have them mark and label a line on paper with whole-number units that are equally spaced and relevant to the addition or subtraction problem.
- Then have them show the addition or subtraction using curved lines segments above the number line and between the numbers marked on the number line. For 49 + 5, start at 49 on the line and draw a curve to 50 Continue drawing curves to 54.
- Drawing the curves or making the "hops" between the numbers will help students focus on a space as the length of a unit and the sum or difference as a length.



Resources/Tools

Rulers
Yardsticks
Meter sticks
Measuring tapes
Cash register tapes or paper strips

See: Progression for Common Core State Standards in Mathematics: K-5, Geometric Measurement for detailed information about this standard.

"Hopping Backward to Solve Problems", NCTM.org, Illuminations. In this lesson students determine differences using the number line to compare lengths.

" Where Will I Land?", NCTM.org, Illuminations. In this lesson students find differences using the number line, a continuous model for subtraction.

Common Misconceptions:

Students may think that they always have to start at zero. Adjustments can be made if measured from a different starting location than zero.

Help students develop and understanding of what the problem is asking. Sometimes "key words" can be misleading. The teaching of a "key word approach" limits the development of understanding what the problems is actually asking.

Domain: Measurement and Data (MD)

Cluster: *Relate addition and subtraction to length.*

Standard: Grade 2.MD.6

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.

Connections: See Grade 2.MD.5

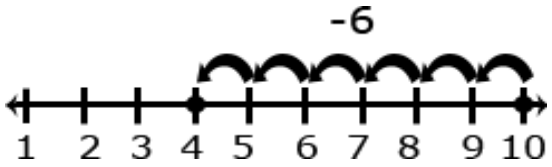
Explanation and Examples:

This standard calls for the student to create number lines within100 to solve addition and subtraction problems. Students should create the number line with evenly spaced points corresponding to the numbers.

Students represent their thinking when adding and subtracting within 100 by using a number line. An interactive whiteboard or document camera can be used to help students demonstrate their thinking. Their thinking should connect to strategies that expand beyond one by one counting.

Example:

$$10 - 6 = 4$$



Instructional Strategies: **See Grade 2.MD.5**

Common Misconceptions: See Grade 2.MD.5

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Measurement and Data (MD)

Cluster: *Work with time and money*

Standard: Grade 2.MD.7

Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections: (2.MD.7-8)

This cluster is connected to:

- Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction and beyond the critical area of focus in addressing, telling time and writing time**.
- *Tell and write time in* Grade 1
- *Represent and solve problems involving addition and subtraction* in Grade 2
- *Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses* of objects in Grade 3.

Explanation and Examples:

This standard calls for students to tell (orally and in writing) and write time after reading analog and digital clocks.

Time should be to 5 minute intervals, and students should also use the terms a.m. and p.m. Teachers should help students make the connection between skip counting by 5s (2.NBT.2) and telling time on an analog clock.

In first grade, students learned to tell time to the nearest hour and half-hour. Students build on this understanding in second grade by skip-counting by 5 to recognize 5-minute intervals on the clock. They need exposure to both digital and analog clocks.

It is important that they can recognize time in both formats and communicate their understanding of time using both numbers and language. Common time phrases include the following: quarter till ___, quarter after ___, ten till ___, ten after ___, and half past ___.

Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies: **(2.MD.7-8)**
Second graders expand their work with telling time from analog and digital clocks to the nearest hour or half-hour in Grade 1 to telling time to the nearest five minutes using a.m. and p.m.

Students should understand that there are 2 cycles of 12 hours in a day - a.m. and p.m. Recording their daily actions in a journal would be helpful for making real-world connections and understanding the difference between these two cycles

An interactive whiteboard or document camera may be used to help students demonstrate their thinking.

Tools/Resources:
Illustrative Mathematics:
Frog and Toad on the number line
Delayed Gratification
Ordering Time

Common Misconceptions: (2.MD.7-8)
Some students might confuse the hour and minutes hands. For the time of 3:45, they say the time is 9:15. Also, some students name the numeral closest to the hands, regardless of whether this is appropriate. For instance, for the time of 3:45 they say the time is 3:09 or 9:03. One way to avoid this confusion is to use Dr. John Van de Walle's strategy of using a one-handed clock to begin telling time. This gets students to focus on the hour hand first. It also helps them understand that the hour hand gives the most significant information when telling time.

Assess students' understanding of the roles of the minute and hour hands and the relationship between them.

Provide opportunities for students to experience and measure times to the nearest five minutes and the nearest hour. Have them focus on the movement and features of the hands on real or geared manipulative clocks.

Domain: Measurement and Data (MD)

Cluster: *Work with time and money*

Standard: Grade 2.MD.8

Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have*?

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2.MD.7

Explanation and Examples:

This standard calls for students to solve word problems involving **either** dollars or cents.

Since students have not been introduced to decimals, problems should either have only dollars or only cents.

Example:
What are some possible combinations of coins (pennies, nickels, dimes, and quarters) that equal 37 cents?

Example:
What are some possible combinations of dollar bills ($1, $5 and $10) that equal 12 dollars?

Since money is not specifically addressed in kindergarten, first grade, or third grade, students should have multiple opportunities to identify, count, recognize, and use coins and bills in and out of context.

They should also experience making equivalent amounts using both coins and bills. “Dollar bills” should include denominations up to one hundred ($1, $5, $10, $20, $100).

Students should solve story problems connecting the different representations. These representations may include objects, pictures, charts, tables, words, and/or numbers. Teachers should make sure that students are using all subtypes of problems from Table 1 from the standards document.

Students should communicate their mathematical thinking and justify their answers. An interactive whiteboard or document camera may be used to help students demonstrate and justify their thinking.

Example:
Sandra went to the store and received 76¢ in change. What are three different sets of coins she could have received?

Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies: **See Grade 2.MD.7**

The topic of money begins at Grade 2 and builds on the work in other clusters in this and previous grades. Help students learn money concepts and solidify their understanding of other topics by providing activities where students make connections between them.

Students use the context of money to find sums and differences less than or equal to 100 using the numbers 0 to 100. They add and subtract to solve one- and two-step word problems involving money situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.

Students use drawings and equations with a symbol for the unknown number to represent the problem. The dollar sign, $, is used for labeling whole-dollar amounts without decimals, such as $29.

Students need to learn the relationships between the values of a penny, nickel, dime, quarter and dollar bill.

Tools/Resources:

Illustrative Mathematics:

Jamir's Penny Jar

Susan's Choice

Visiting the Arcade

Saving Money 1

Alexander, Who Used to be Rich Last Sunday

Pet Shop

Choices, Choices, Choices

Common Misconceptions:

Sometimes students will record twenty-nine dollars as 29$. Remind them that the dollar sign goes in front. The cent sign goes after the number and there is no decimal point used with the cent sign nor can both signs be used in the same amount.

Students might over-generalize the value of coins when they count them. They might count them as individual objects. Also some students think that the value of a coin is directly related to its size, so the bigger the coin, the more it is worth.

Place pictures of a nickel on the top of five-frames that are filled with pictures of pennies. In like manner, attach pictures of dimes and pennies to ten-frames and pictures of quarters to 5 x 5 grids filled with pennies. Have students use these materials to determine the value of a set of coins in cents.

Domain: Measurement and Data (MD)

Cluster: *Represent and interpret data*

Standard: Grade 2.MD.9

Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: (2.MD.9-10)

This cluster goes beyond the Second Grade Critical Area of Focus to address, using data representations.
This cluster connects to:

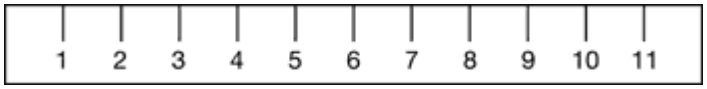
- *Measure lengths indirectly and by iterating length units* and *Represent and interpret data* in Grade 1
- *Represent and interpret data* in Grade 3.

Explanation and Examples:

This standard calls for students to represent the length of several objects by making a line plot. Students should round their lengths to the nearest whole unit.

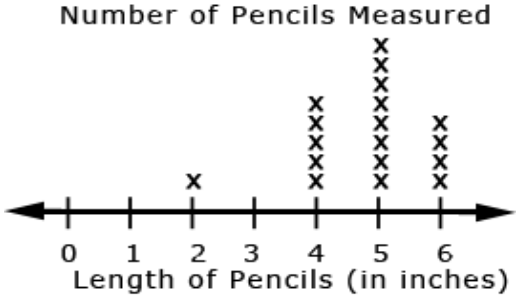
Example:

Measure objects in your desk to the nearest inch, display data collected on a line plot. How many objects measured 2 inches? 3 inches? Which length had the most number of objects? How do you know?



This standard emphasizes representing data using a line plot. Students will use the measurement skills learned in earlier standards to measure objects. Line plots are first introduced in this grade level.

A line plot can be thought of as plotting data on a number line. An interactive whiteboard may be used to create and/or model line plots as well as "Class" line-plots on chart paper.



Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies: **(2.MD.9-10)**

Line plots are useful tools for collecting data because they show the number of things along a numeric scale.

The line plot is made by simply drawing a number line then placing an X above the corresponding value on the line that represents each piece of data. Make sure students understand that the Xs need to be of consistent size and are lined up similar to a bar graph.

Line plots are essentially bar graphs with a potential bar for each value on the number line but generally are quicker to make which also allows for more depth of instruction when less time is used for the "drawing/shading" of a bar graph. It also reinforces the ideas presented on a number line. Students need to make sure their Xs are all the same size so their line plot is not distorted.

Pose a question related to the lengths of several objects. Measure the objects to the nearest whole inch, foot, centimeter or meter. Create a line plot with whole-number units (0, 1, 2, ...) on the number line to represent the measurements.

At first students should create real object or picture graphs (pictograph) (where the object is drawn rather than a number). On picture graphs record the number of countable parts.

These graphs show items in a category and do not have a numerical scale. For example, a real object graph could show the students' shoes (one shoe per student) lined end to end in horizontal or vertical rows by their color.
Students would simply count to find how many shoes are in each row or bar. The graphs should be limited to 2 to 4 rows or bars.

Students then move to making horizontal or vertical bar graphs with two to four categories and a single-unit scale. Use the information in the graphs to pose and solve simple put together, take-apart, and compare problems illustrated in Table 1 (Appendix).

Tools/Resources:

Illustrative Mathematics:
The Longest Walk
Growing Bean Plants
Hand Span Measures

Common Misconceptions: (2.MD.9-10)

The attributes for the same kind of object can vary. This will cause equal values in an object graph to appear unequal. For example, when making an object graph using shoes for boys and girls, five adjacent boy shoes would likely appear longer than five adjacent girl shoes. To standardize the objects, place the objects on the same-sized construction paper or sticky-note, then make the object graph.

Domain: Measurement and Data (MD)

Cluster: *Represent and interpret data*

Standard: Grade 2.MD.10

Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2.MD.9

Explanation and Examples:

This standard calls for students to work with categorical data by organizing, representing and interpreting data. Students should have experiences posing a question with 4 possible responses and then work with the data that they collect.

Example:

Students pose a question and the 4 possible responses. Which is your favorite flavor of ice cream? Chocolate, vanilla, strawberry, or cherry?

Students collect their data by using tallies or another way of keeping track.

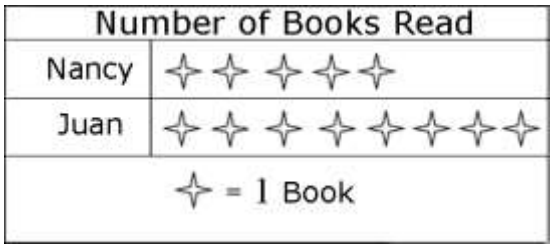
Students organize their data by totaling each category in a chart or table. Picture and bar graphs are introduced in Second Grade.

Flavor	Number of People
Chocolate	12
Vanilla	5
Strawberry	6
Cherry	9

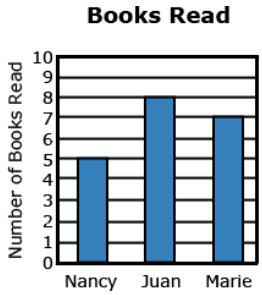
Students display their data using a picture graph or bar graph using a single unit scale. Students answer simple problems related to addition and subtraction that ask them to put together, take apart, and compare numbers. (See Appendix Table 1, page 67) for examples of these.

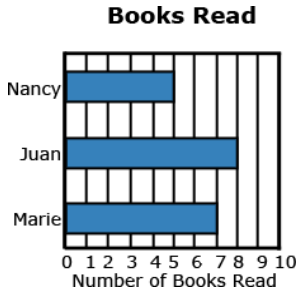
Students should draw both picture and bar graphs representing data that can be sorted up to four categories using single unit scales (e.g., scales should count by ones). The data should be used to solve put together, take-apart, and compare problems as listed in Table 1, page 49.

In second grade, picture graphs (pictographs) include symbols that represent single units. Pictographs should include a title, categories, category label, key, and data.



Second Graders should draw both horizontal and vertical bar graphs. Bar Graphs include a title, scale, scale label, category label and data.





Instructional Strategies: **See Grade 2.MD.9**

Tools/Resources:

Favorite Ice Cream Flavor

See: "Pies for Sale," NCSM, Great Tasks for Mathematics K-5, (2013).

"Barn Yard Legs", Georgia Department of Education. This activity uses children's literature connections and has students create graph and tables and interpret their meaning.

Common Misconceptions: See Grade 2.MD.9

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Geometry (G)

Cluster: *Reason with shapes and their attributes.*

Standard: Grade 2.G.1

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. *Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections:

This cluster is connected to:

- Second Grade Critical Area of Focus #4**, Describing and analyzing shapes**.
- *Reason with shapes and their attributes* in Grade 1
- *Develop understanding of fractions as numbers* and *Reason with shapes and their attributes* in Grade 3.

Explanation and Examples:

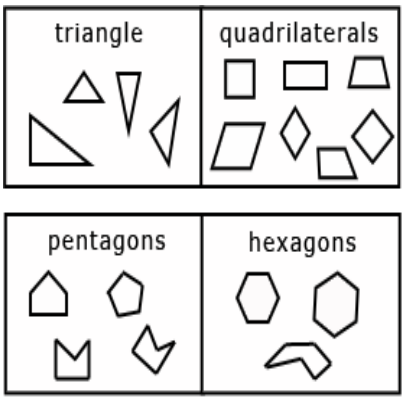
This standard calls for students to identify (recognize) and draw shapes based on a given set of attributes. These include triangles, quadrilaterals (squares, rectangles, and trapezoids), pentagons, hexagons and cubes.

Example:

Draw a closed shape that has five sides. What is the name of the shape? Student - I drew a shape with 5 sides. It is called a pentagon.

Students identify, describe, and draw triangles, quadrilaterals, pentagons, and hexagons. Pentagons, triangles, and hexagons should appear as both regular (equal sides and equal angles) and irregular.

Students recognize all four sided shapes as quadrilaterals. Students use the vocabulary word “angle” in place of “corner” but they do not need to name angle types.



Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies: **(2.G.1-3)**
Geosticks, geoboards, interactive whiteboards and document cameras may be used to help identify shapes and their attributes. Shapes should be presented in a variety of orientations and configurations.

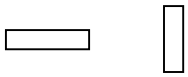
Tools/Resources:
Polygons

See: "Creative Cards", NCSM, Great Tasks for Mathematics K-5, (2013).

"3-D Detectives", Georgia Department of Education. In this activity students describe and classify plane figures (triangles, square, rectangle, trapezoid, quadrilateral, pentagon, hexagon, and irregular polygonal shapes).

"What's in a Name", Georgia Department of Education. In this activity students will describe and classify plane figures (triangles, square, rectangle, trapezoid, quadrilateral, pentagon, hexagon, and irregular polygonal shapes) according to the number of edges and vertices.

Common Misconceptions: (2.G.1-3)
Some students may think that a shape is changed by its orientation. They may see a rectangle with the longer side as the base, but claim that the same rectangle with the shorter side as the base is a different shape. This is why is it so important to have young students handle shapes and physically feel that the shape does not change regardless of the orientation, as illustrated below.



If students are only shown equilateral triangles then when they see scalene or isosceles triangles, they do not recognize them as triangles even though they have three sides. So you must make sure you are always showing students various types of shapes and not just the regular shapes that they see in pattern blocks and on posters.

Domain: Geometry (G)

Cluster: *Reason with shapes and their attributes.*

Standard: Grade 2. G.2

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

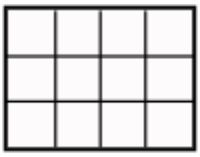
Connections: See Grade 2. G.1

Explanation and Examples:

This standard calls for students to partition a rectangle into squares (or square-like regions) and then determine the total number of squares. This relates to the standard 2.OA.4 where students are arranging objects in an array of rows and columns.

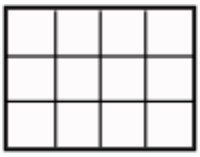
Example:

Split the rectangle into 3 rows and 4 columns. How many small squares did you make?



This standard is a precursor to learning about the area of a rectangle and using arrays for multiplication. An interactive whiteboard or manipulatives such as square tiles or other square shaped objects can be used to help students' partition rectangles.

Rows are horizontal and columns are vertical.



Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies: **See Grade 2. G.1**

Modeling multiplication with partitioned rectangles promotes students' understanding of multiplication. Tell students that they will be drawing a square on grid paper. The length of each side is equal to 2 units.

- Ask them to guess how many 1 unit by 1 unit squares will be inside this 2 unit by 2 unit square.
- Students now draw this square and count the 1 by 1 unit squares inside it.
- They compare this number to their guess.
- Next, students draw a 2 unit by 3 unit rectangle and count how many 1 unit by 1 unit squares are inside.
- Now they choose the two dimensions for a rectangle, predict the number of 1 unit by 1 unit squares inside, draw the rectangle, count the number of 1 unit by 1 unit squares inside and compare this number to their guess.
- Students repeat this process for different-size rectangles. Finally, ask them to what they observed as they worked on the task.

It is vital that students understand different representations of fair shares. Provide a collection of different-size circles and rectangles cut from paper. Ask students to fold some shapes into halves, some into thirds, and some into fourths. They compare the locations of the folds in their shapes as a class and discuss the different representations for the fractional parts.

To fold rectangles into thirds, ask students if they have ever seen how letters are folded to be placed in envelopes. Have them fold the paper very carefully to make sure the three parts are the same size. Ask them to discuss why the same process does not work to fold a circle into thirds.

Common Misconceptions: See Grade 2. G.1

Domain: Geometry (G)

Cluster: *Reason with shapes and their attributes.*

Standard: Grade 2. G.3

Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2.G.1

Explanation and Examples:

This standard calls for students to partition (divide) circles and rectangles into 2, 3 or 4 equal shares (regions).

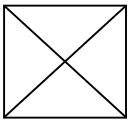
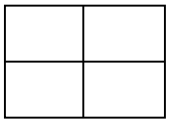
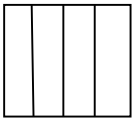
Students should be given ample experiences to explore this concept with paper strips and pictorial representations.

Students should also work with the vocabulary terms halves, thirds, half of, third of, and fourth (or quarter) of. While students are working on this standard, teachers should help them to make the connection that a "whole" is composed of two halves, three thirds, or four fourths.

This standard also addresses the idea that equal shares of identical wholes may not have the same shape.

Example:

Divide each square into fourths a different way.

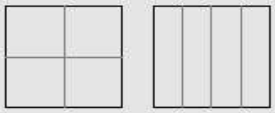




This standard introduces fractions in an **area model**. Students need experiences with different sizes, circles, and rectangles. These different partitions of a square afford the opportunity for students to identify correspondences between the differently-shaped fourths (MP.1), and to explain how one of the fourths shown in these squares can be transformed into one of the "other" fourths shown. (MP.7).

Different pattern blocks compose a regular hexagon

2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Squares partitioned into fourths



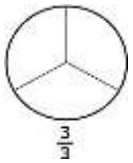
These different partitions of a square afford the opportunity for students to identify correspondences between the differently-shaped fourths (MP.1), and to explain how one of the fourths on the left can be transformed into one of the fourths on the right (MP.7).

2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

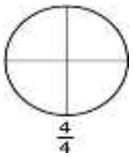
See Learning Progressions on Geometry for dialed information: http://commoncoretools.files.wordpress.com/2012/06/ccss_progression_g_k6_2012_06_27.pdf)

For example, students should recognize that when they cut a circle into three equal pieces, each piece will equal one third of its original whole.

In this case, students should describe the whole as three thirds. If a circle is cut into four equal pieces, each piece will equal one fourth of its original whole and the whole is described as four fourths.

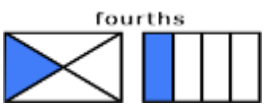


$\frac{3}{3}$



$\frac{4}{4}$

It is important for students to see circles and rectangles partitioned in multiple ways so they learn to recognize that equal shares can be different shapes within the same whole. An interactive whiteboard may be used to show partitions of shapes.



Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies: **See Grade 2.G.1**

Tools/Resources:

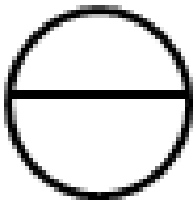
Illustrative Mathematics:

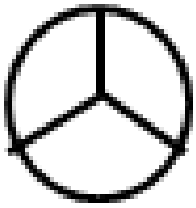
Representing Half of a Rectangle

Which Pictures Represent One Half?

Common Misconceptions: See Grade 2.G.1

Students also may believe that a region model represents one out of two, three or four fractional parts without regard to the fact that the parts have to be equal shares, e.g., a circle divided by two equally spaced horizontal lines represents three thirds.





Appendix

Major Supporting Additional **Depth Opportunities(DO)**

TABLE 1. Common Addition and Subtraction Situations[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together / Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ or $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$ or $5 = 5 + 0$ $5 = 1 + 4$ or $5 = 4 + 1$ $5 = 2 + 3$ or $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ or $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ or $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ or $? + 3 = 5$

[1]These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common Multiplication and Division Situations[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ And $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*: You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*: You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*: You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] **Area**[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*: A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*: A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*: A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

TABLE 3. The Properties of Operations

Associative property of addition	$(a+b)+c=a+(b+c)$
Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
Existence of additive inverses	For every (a) there exists$(-a)$ so that $a+(-a)=(-a)+a=0$
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Existence of multiplicative inverses	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
Distributive property of multiplication over addition	$a\times(b+c)=a\times b+a\times c$

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

TABLE 4. The Properties of Equality*

Reflexive property of equality	$a=a$
Symmetric property of equality	If $a=b$ then $b=a$
Transitive property of equality	If $a=b$ and $b=c$, then $a=c$
Addition property of equality	If $a=b$ then $a+c=b+c$
Subtraction property of equality	If $a=b$ then $a-c=b-c$
Multiplication property of equality	If $a=b$ then $a\times c=b\times c$
Division property of equality	If $a=b$ and $c\neq 0$ then $a\div c=b\div c$
Substitution property of equality	If $a=b$ then b may be substituted for a in any expression containing a.

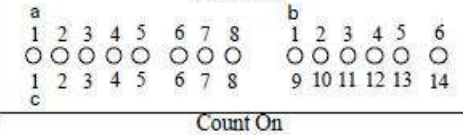
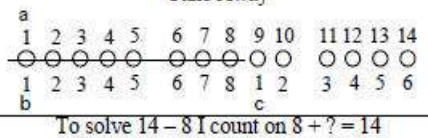
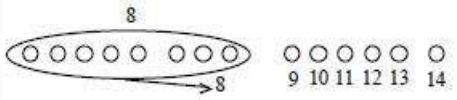
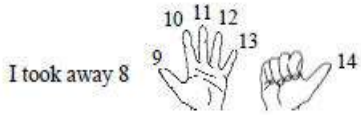
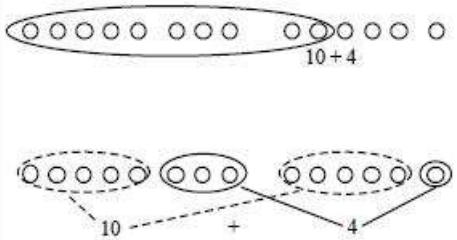
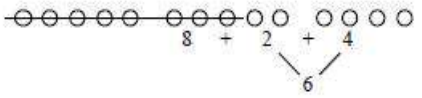
Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a<b, a=b, a>b$.
If $a>b$ and $b>c$ then $a>c$
If $a>b$ then $b<a$
If $a>b$ then $-a<-b$
If $a>b$ then $a\pm c>b\pm c$
If $a>b$ and $c>0$ then $a\times c>b\times c$
If $a>b$ and $c<0$ then $a\times c<b\times c$
If $a>b$ and $c>0$ then $a\div c>b\div c$
If $a>b$ and $c<0$ then $a\div c<b\div c$

Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

TABLE 6. Development of Counting in K-2 Children

Levels	8 + 6 = 14	14 – 8 = 6
Level 1: Count all	Count All 	Take Away 
Level 2: Count on	Count On 	To solve 14 – 8 I count on 8 + ? = 14  8 to 14 is 6 so 14 – 8 = 6
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend Make a ten (from 5's within each addend)	Recompose: Make a Ten 	14 – 8: I make a ten for 8 + ? = 14 
Doubles ± *n*	6 + 8 = 6 + 6 + 2 = 12 + 2 = 14	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Beginning--A child can count very small collections (1-4) collection of items and understands that the last word tells "how many" even. Beyond on small numbers the number words may be said without the lack of numerical understanding. This is often referred to as rote counting.

Level 1—The child uses one to one correspondence to find the number of objects in two sets. Even if the quantity is known for the first set, the child will start with the first set and continue counting on into the second set. The child begins the count with one. This also connects to Piaget's Hierarchical Inclusion – that in order to have 7 – you have to have 6, 5, 4, etc.

Level 2 – At this level the student begins the counting, starting with the known quantity of the first set and "counts on" from that point in the sequence to establish how many. This method is used to find the total in two sets. This counting is not rote. This level of counting requires several connections between cardinality and counting meanings of the number words

Table 7. Cognitive Rigor Matrix/Depth of Knowledge (DOK)

Common Core State Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

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